

基礎統計 2004

数式ばかりで見にくいです。間違いを見つけたら知らせてください。

問一

$$(1) () N(50, 10^2)$$

$$\therefore P(70 \leq X) = P(2 \leq Z) = 0.023$$

$$() P(40 \leq X \leq 60) = P(-1 \leq Z \leq 1) = 0.683$$

$$() P(X \leq 55) = P(Z \leq 0.5) = 1 - Q(0.5) = 0.739$$

$$(2) N(3.2, 0.4^2) \quad E(\bar{X}) = 3.2 \quad V(\bar{X}) = 0.01 \quad \therefore \mu = 3.2 \quad \sigma = 0.1$$

$$P(\bar{X} \leq 3.35) = P(Z \leq 1.5) \cong 9.33$$

$$(3) E(X) = \sum_{x=1}^N x \cdot \frac{1}{N} = \frac{1}{N} \cdot \frac{1}{2} N(N+1) = \frac{1}{2}(N+1)$$

$$E(X^2) = \sum_{x=1}^N x^2 \cdot \frac{1}{N} = \frac{1}{6}(N+1)(2N+1)$$

$$\therefore V(X) = E(X^2) - \{E(X)\}^2 = \frac{1}{12}(N+1)(N-1)$$

問二

$$(1) {}_6C_2(0.3)^2(0.7)^4 = 0.324$$

$$(2) 1000 \cdot 0.002 = 2 \text{ なので } X \sim Po(2)$$

$${}_{1000}C_3(0.2)^3(1-0.2)^{1000-3} \rightarrow e^{-2} \frac{2^3}{3!} = 0.183$$

$$(3) X \sim Ge(0.4)$$

$$P(X \leq 3) = \sum_{x=1}^3 0.4(0.6)^{x-1} = 0.784$$

問三

$$(1) () P(X = x) = \frac{{}^{10000}C_x}{2^{10000}} (x = 0, 1, \dots, 10000) \text{ であり}$$

$$\mu = 5000, \sigma^2 = 10000 \cdot \frac{1}{4} = 2500, \sigma = 50$$

$$P(4850 \leq X \leq 5150) = P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) \approx 0.889$$

$$() X \sim Bi(10000, 1/2) \quad \bar{X} \sim N(5000, 2500)$$

$$Z = \frac{X - 5000}{50} \sim N(0,1) \quad P(4850 \leq X \leq 5150) = P(-3 \leq Z \leq 3) = 0.997$$

$$(2) \bar{X} \sim N\left(p, \frac{p(1-p)}{n}\right) \text{なので } P\left(-1.96 \leq \frac{\bar{X} - p}{\sqrt{p(1-p)/n}} \leq 1.96\right) = 0.95$$

$p(1-p) = \bar{X}(1-\bar{X})$ と近似できるので $\bar{X} = 0.7, n = 100$ を代入すると

$$0.61 \leq p \leq 0.79$$

問四

$$(1) X_1 \cdots X_{10} \sim (\text{独立})N(\mu_1, \sigma_1^2) \text{より } \frac{\bar{X} - \mu_1}{\sqrt{s_1^2/10}} \sim t(9)$$

$$P\left(-2.262 \leq \frac{\bar{X} - \mu}{\sqrt{s^2/10}} \leq 2.262\right) = 0.95 \quad \bar{X} - 2.262\sqrt{s^2/10} \leq \mu \leq \bar{X} + 2.262\sqrt{s^2/10}$$

$$12.82 \leq \mu_1 \leq 15.98$$

$$(2) \frac{(n-1)s_1^2}{\sigma_1^2} \sim \chi^2(n-1) \text{だから } \frac{44.1}{\sigma_1^2} \sim \chi^2(9)$$

$$\therefore P\left(2.70 \leq \frac{44.1}{\sigma^2} \leq 19.02\right) = 0.95 \text{ これを解いて } 2.32 \leq \sigma^2 \leq 16.33$$

(3)(4)

問五

$$(1) b = \frac{C_{xy}}{S_x^2} = 0.311 \quad a = \bar{y} - b\bar{x} = 120.45 \quad \therefore y = 120.4x + 0.311$$

(2) 親の身長と子の身長の間には正の相関関係がある。

$$(3) \text{決定係数} = r^2 = \left(\frac{C_{xy}}{S_x S_y}\right)^2 = \frac{144}{38.7 \cdot 31.8} = 0.117$$